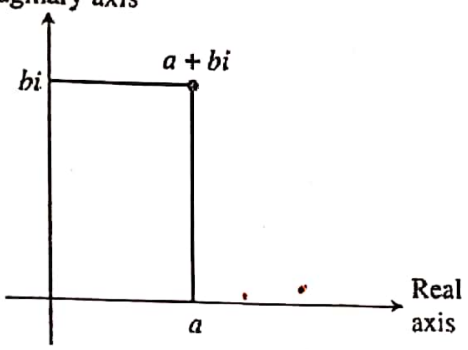
Section 6.6 NOTES DeMoivre's Theorem and nth roots

The Complex Plane:



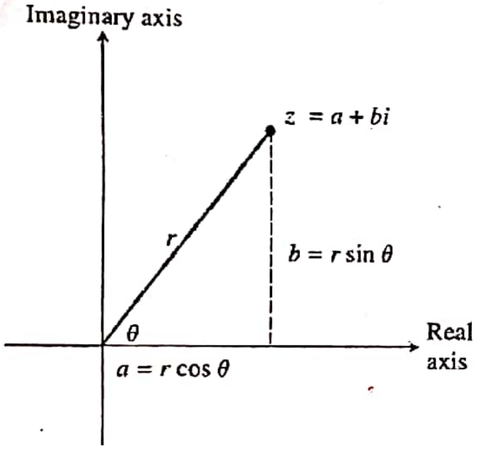


FIGURE 6.59 If r is the distance of $z = a + bi$ from the origin and θ is the directional angle shown, then $z = r(\cos \theta + i \sin \theta)$, which is the polar form of z.

ABSOLUTE VALUE of a Complex # (Modulus) $|z| = \sqrt{a^2+b^2}$

DEFINITION Polar Form of a Complex Number

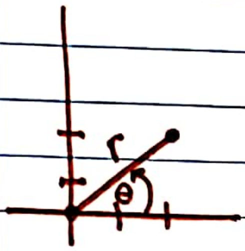
The **polar form** of the complex number $z = a + bi$ is

$$z = r(\cos \theta + i \sin \theta)$$

where $a = r \cos \theta$, $b = r \sin \theta$, $r = \sqrt{a^2 + b^2}$, and $\tan \theta = b/a$. The number r is the *absolute value* or *modulus* of z, and θ is an **argument** of z.

Writing Complex Numbers in Polar form

1) $2 + 2i$



$r = \sqrt{2^2 + 2^2}$

$\sqrt{8}$

$2\sqrt{2}$

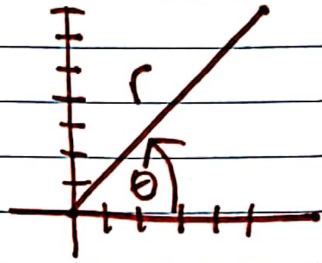
$\theta = \tan^{-1}(1)$

$45° / \pi/4$

$$\boxed{z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$$

2) $5 + \sqrt{-49}$

$5 + 7i$



$r = \sqrt{5^2 + 7^2}$

$\sqrt{25 + 49} = \sqrt{74}$

$\theta = \tan^{-1}\left(\frac{7}{5}\right)$

$54.5°$

0.951 rad

$$\boxed{z = \sqrt{74}\left(\cos 54.5° + i\sin 54.5°\right)}$$

or

$$z = \sqrt{74}\left(\cos 0.951 + i\sin 0.951\right)$$

ODDS - Notes

EVENS (on lined paper) - HW

In Exercises 13–18, write the complex number in standard form $a + bi$. (rectangular form)

UNIT CIRCLE (UC)

13. $3(\cos 30° - i \sin 30°)$

HW 14. $8(\cos 210° + i \sin 210°)$ UC

15. $5[\cos(-60°) + i \sin(-60°)]$

HW 16. $5\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$ UC

17. $\sqrt{2}\left(\cos\frac{7\pi}{6} + i \sin\frac{7\pi}{6}\right)$

HW 18. $\sqrt{7}\left(\cos\frac{\pi}{12} + i \sin\frac{\pi}{12}\right)$ calc

13) $3\left(\frac{\sqrt{3}}{2} - i \cdot \frac{1}{2}\right) = \boxed{\frac{3\sqrt{3}}{2} - \frac{3}{2}i}$

15) $5\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = \boxed{\frac{5}{2} - \frac{5\sqrt{3}}{2}i}$

17) $\sqrt{2}\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = \boxed{-\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i}$